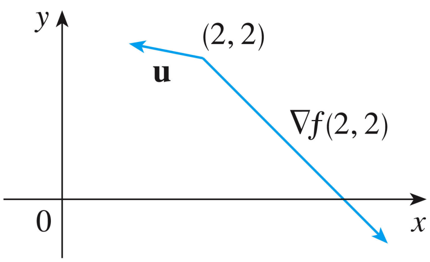
Lesson 17. Maximizing the Directional Derivative

0 Warm up

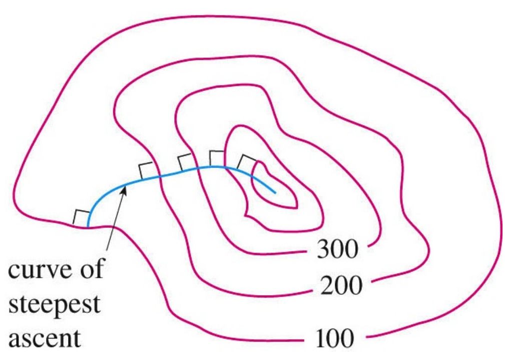
Example 1. Use the figure below to estimate $D_{\vec{u}} f(2,2)$. Assume $|\nabla f(2,2)| \approx 3$, and the angle between $\nabla f(2,2)$ and $\vec{u}$ is approximately $3\pi/4$.



1 Maximizing the directional derivative

- From the previous lesson: in words, the directional derivative of f at (x,y) in the direction of unit vector $\vec{u}$ is

- Questions:
  - In which direction does f change the fastest? (steepest ascent or descent)
  - What is this maximum rate of change?
- **Important theorem:** (f is a function of 2 or 3 variables)
  - The maximum value of $D_{\vec{u}} f$ is $|\nabla f|$
  - The maximum value occurs when $\vec{u}$ is in the same direction as ∇f



- As a result, the gradient is

2 Examples

Example 2. Let $f(x, y) = xe^y$.

a. Find the rate of change of f at the point $P(2, 0)$ in the direction from P to $Q(\frac{1}{2}, 2)$.
b. In what direction does f have the maximum rate of change? What is this maximum rate of change?

Example 3. Find the directional derivative of $f(x, y) = \sqrt{xy}$ at $P(2, 8)$ in the direction of $Q(5, 4)$.

Example 4. Let $f(x, y, z) = \sqrt{x^2 + y^2 + z^2}$. Find the maximum rate of change of f at $(3, 6, -2)$ and the direction in which it occurs.

Example 5. Find all points at which the direction of fastest change of the function $f(x, y) = x^2 + y^2 - 2x - 4y$ is $\vec{i} + \vec{j}$. *Hint.* Your answer should be an equation in x and y.